HEAT EXCHANGE IN THE INITIAL SECTION OF A PIPE WITH LAMINAR FLOW IN THE BOUNDARY LAYER AND A CONSTANT WALL TEMPERATURE

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Equations are proposed for calculating the local and average coefficients of heat exchange in the dynamic initial section of a round pipe with laminar flow in the boundary layer and a constant wall temperature.

In [1] it is recommended that the local heat exchange in the dynamic initial sections of round pipes with mixed flow of an airstream and a constant wall temperature be calculated by enlisting, for the zone with a laminar boundary layer, an equation similar in structure to Pohlhausen's equation for a longitudinally bathed plate [2]:

$$\mathrm{Nu}_X = 0.321\mathrm{Re}_X^{0.5}. \tag{1}$$

Such an approach seems physically justified for the region of the dynamic initial section with uniform velocity fields in the predominant part of the cross section at a small pressure gradient. In contrast to a plate, however, the intensity of heat exchange in pipes stabilizes with greater distance and, therefore, at high values of the first critical Reynolds number, which characterizes the start of the transition in the boundary layer, the dependence (1) should lead to decreased Nusselt numbers. The aim of the present investigation is to obtain data on local heat exchange with laminar flow and t_w = const in the dynamic initial section of a pipe in the region of $\mathrm{Re}_X > 10^5$.

The local coefficients of heat exchange were measured in a pipe with an inner diameter of 36 mm and a length of 2000 mm. It is made up of six sections, independently heated by boiling water, with a heated entrance nozzle profiled with a radius of curvature R/d = 1.0. The pipe is connected to the suction of a blower, as a result of which a constant low degree of turbulence of the air, supplied to the opposite part of the stand from the space of the laboratory room, is maintained ahead of the nozzle (ε = 0.1-0.3%). One of the linear sections 100 mm long with six heat-flux pickups [3] built into its inner surface serves for the determination of the local coefficients of heat exchange. Transposition of the linear sections provides the possibility of determining the local coefficients of heat exchange with a step of 10-25 mm in the axial direction.

A diagram of the entrance section of the stand with the measurement section located directly behind the nozzle is shown in Fig. 1. A coordinate spacer 6 equipped with a Pitot tube 10 with a diameter of 0.9 mm and a thermocouple 7 with a shield 1.0 mm in diameter is adjacent to the measurement section 2. The longitudinal axis of the stand is inclined at a 45° angle to the horizontal in order to facilitate the natural circulation of water in the heating systems of the linear sections. Water is circulated by a pump through the inner cavity of the nozzle from the water jacket of the nearest linear section. In the working mode the wall temperature along the length of the flowing section of the stand is kept constant with a deviation not exceeding ±1.0°C.

The mean flow-rate velocity and temperature of the stream at the exit from the measurement section are calculated from the results of measurements of the velocity and temperature fields. In the cross sections where the pickups are located the mean flow-rate temperatures are calculated from the heat balance. A more detailed description of the stand and the method of initial treatment of the experimental material are presented in [4].

The experiments were conducted within the following limits of the controlling parameters: W = 6-70 m/sec; $\mathrm{Re}_d = (10\text{-}160)\cdot 10^3$; t_w = 94-99°C; t_{in} = 18-20°C. The calculated maximum error of the local coefficients of heat exchange is 7% for the upper and 19% for the lower limits on the velocity.

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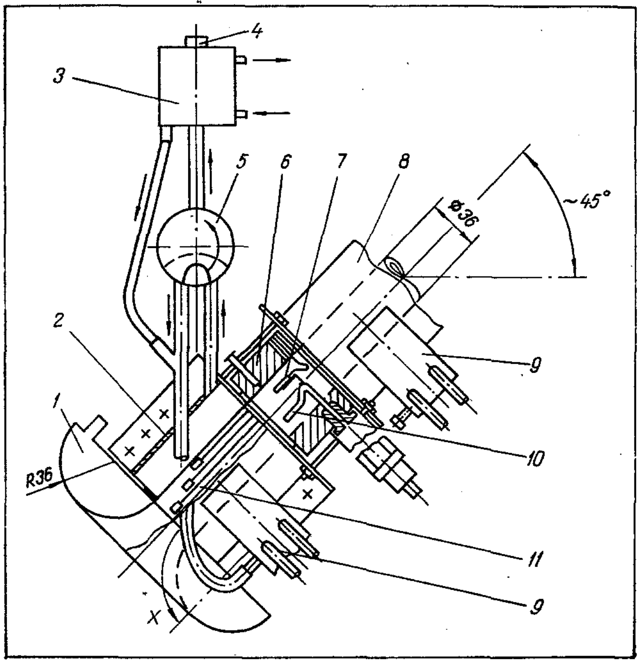

Fig. 1. Construction of entrance section of the stand with the measurement section and coordinate spacer: 1) entrance nozzle with a constriction of 9; 2) measurement section; 3) condenser; 4) safety valve; 5) separator; 6) housing of coordinate spacer; 7) thermocouple of coordinate spacer; 8) linear section; 9) electric heater; 10) Pitot tube; 11) built-in heat-flux pickups.

Distributions of the local coefficients of heat exchange along the X coordinate, reckoned from the start of the constriction, are shown in Fig. 2 for several constant stream velocities. The curves of $\alpha_{loc} = f(X)$ have the usual form for mixed flow. The extended region with a laminar boundary layer and a reduced heat-exchange intensity reaches a length of 1100–1200 mm as $Re_d \to 10^4$. The dashed lines in Fig. 2 are plotted in accordance with Eq. (2b). They unite the results of measurements in the region with a turbulent boundary layer with a scatter of no more than ±8%. Equation (2b) follows from Eq. (2a), proposed in [5] for the calculation of local coefficients of heat exchange with developed turbulent flow in pipes and X/d > 15, with Pr = 0.71:

$$Nu_{d,X} = 0.022\,Re_{d,X}^{0.8}\,Pr^{0.43}, \tag{2a}$$

$$Nu_X = 0.0189\,Re_X^{0.8}\left(\frac{X}{d}\right)^{0.2}. \tag{2b}$$

The graphs of Fig. 2 allow one to establish rather clearly the position of the lower boundary of the transition zone, since the dependence $\alpha_{loc} = f(X)$ undergoes a sharp break here (8 — points to which the critical coordinates are referred). The Reynolds number $Re_{X_{cr}}^{II}$ proves to be constant and comprises $740 \cdot 10^3 \pm 10\%$. The situation is less definite regarding the estimation of the first critical coordinate, especially at low Re_d. Therefore, the quantity $Re_{X_{cr}}^{I}$ was determined on the basis of an analysis of the variation in the thickness of the dynamic boundary layer along the length of the pipe.

The velocity (and temperature) fields were determined in the experiments in the outer part of the boundary layer at distances from the wall exceeding 0.5 mm and with an interval of 100 mm along the longitudinal coordinate starting with the cross section X/d = 4.44. In the region with developed turbulent flow (X/d > 35) with the use of the Blasius friction law the minimum dimensionless distances from the wall were $Y^+ \geq 30$. The thickness of the dynamic boundary layer was determined from the graphs of W = f(Y) as the distance from the heat-transfer surface to the point where the stream velocity coincides with the velocity at the pipe axis with an accuracy no worse than 0.5%.

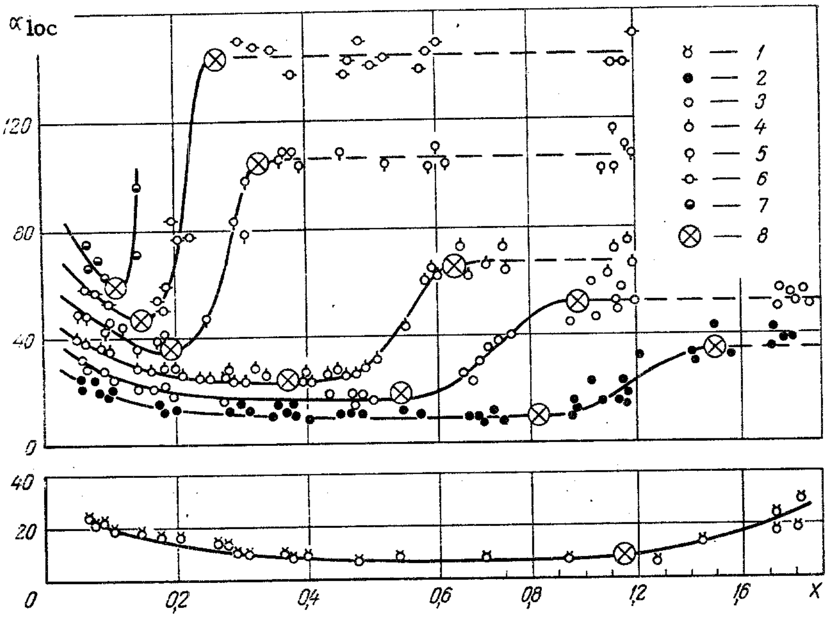

Fig. 2. Results of determination of local coefficients of heat exchange in the initial section of a pipe: 1) Re_d = $(1.1-1.45)\cdot 10^4$; 2) $(1.85-2.05)\cdot 10^4$; 3) $(2.9-3.05)\cdot 10^4$; 4) $(4.1-4.4)\cdot 10^4$; 5) $(7.7-8.4)\cdot 10^4$; 6) $(1.07-1.11)\cdot 10^5$; 7) $1.57\cdot 10^5$; 8) transition points. α_{loc}, $W/m^2\cdot deg$; X, m.

The interpretation of the results of an analysis of the velocity fields in the coordinates $\delta_W/X = f(Re_X)$ proved to be the most convenient for an estimate of the first critical Reynolds number. The left-hand part of the logarithmic graphs of $\delta_W/X = f(Re_X)$, shown in Fig. 3a, is linear. As Re_d decreases the steepness of the dependence $\delta_W/X = f(Re_X)$ grows, and at the lower limit of Reynolds numbers it coincides almost exactly with a slope of -0.5. Passing a minimum at $Re_X = Re^{I}_{Xcr} = 440\cdot 10^3 \pm 5\%$, the parameter δ_W/X begins to increase rapidly. At the maximum on the right $\delta_W = 0.5d$. Then δ_W = const and the graphs of $\delta_W/X = f(Re_X)$ again become linear with a slope of -1.*

The coordinates of the start of the transition found from Fig. 3a, are located directly ahead of the rising section of the curves of $\alpha_{loc} = f(X)$ in Fig. 2, with the ratio X''_{cr}/X'_{cr} being close to 1.7, which is in satisfactory agreement with the known data for a longitudinally bathed plate [7].

The velocity profiles in the region with a laminar boundary layer proved to be fuller, and the thickness of the dynamic boundary layer smaller, than follows from Langhaar's equations for isothermal flow [8]. This is shown in Fig. 3b, where the data of Fig. 3a, which pertain to laminar flow, are analyzed in the system of coordinates $\delta_W/d = f(1/Re_d\cdot X/d)$. In the interval of Reynolds numbers $Re_d = (20-80)\cdot 10^3$ and relative lengths $X/d = 4.4-35.0$, where laminar modes of flow could be traced, the experimental material is united by curve 2. At $1/Re_d\cdot X/d > 2\cdot 10^{-4}$ the experimental values of δ_W/X are 20-30% lower than those calculated from the universal dependence of Langhaar (curve 1).

Such pronounced disagreements with the results of calculations in accordance with Langhaar are explained by the nonisothermicity of the flow under the experimental conditions and by the specific action on the stream of the entrance nozzle, where the flow develops under conditions of constriction. The latter is evidently the most important. As shown in [9] where the velocity distributions in a constricting channel are analyzed on the basis of an exact solution, the velocity profiles are fuller in this case than in a channel with parallel walls. Therefore, behind the nozzle, in the initial sections of the pipe itself, the thickness of the dynamic boundary layer grows more slowly than would be expected from a calculation by the equations of [8]. The experimental results indicate that this tendency is retained

*The curves of $\delta_W/X = f(Re_X)$ serve as confirmation of the qualitative conclusions of Petukhov and Yushin, based on the results of visual observations of the behavior of a colored jet of liquid in the dynamic initial section of a pipe with mixed flow [6].

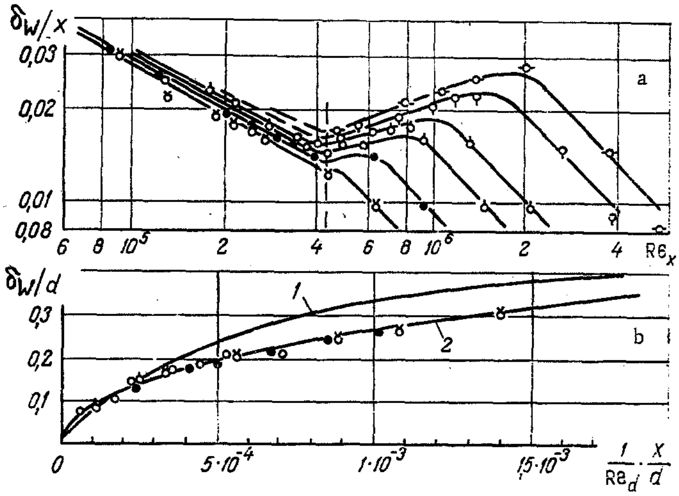

Fig. 3. Results of a study of velocity fields in the initial section of a pipe (notation same as in Fig. 2): a) dependence $\delta_W/X = f(Re_X)$; b) dependence $\delta_W/d = f(1/Re_d \cdot X/d)$; 1) based on experimental data; 2) calculation according to [8].

in subsequent sections for a rather long time, and the velocity profiles may be transformed into the parabolic distributions which are usual for laminar flow only at a considerable distance from the entrance.

In Fig. 4 the entire mass of experimental data on local heat exchange in the dynamic initial section is analyzed in the coordinates $Nu_d = f(1/Pe \cdot X/d)$. The dashed lines are calculated from Eq. (2b). Curve II is constructed from an equation which is a consequence of simple transformations of Eq. (1):

$$\mathrm{Nu}_d = 0.381\left(\frac{1}{\mathrm{Pe}}\cdot\frac{X}{d}\right)^{-0.5}. \tag{3}$$

In the region with laminar flow in the boundary layer the experimental points form a family of curves pertaining to constant Reynolds numbers. At small X/d the local Nusselt numbers decrease monotonically along the abscissa, being joined by the envelope curve I at all velocities. When the reduced lengths become less than 10^{-4} curve I asymptotically approaches a slope of −0.5, while the values of Nu_d coincide with those calculated from Eq. (3) with an accuracy of ±8%. In the interval of $10^{-5} < 1/Pe \cdot X/d < 10^{-3}$ curve I is approximated by the equation

$$\mathrm{Nu}_d = 0.381\left(\frac{1}{\mathrm{Pe}}\cdot\frac{X}{d}\right)^{-0.5} + 2.3. \tag{4}$$

The branching of the experimental data on the Reynolds number occurs at distances of X/d = 2-4 from the entrance and is accompanied by intense oscillations in the local heat fluxes which increase downstream and whose amplitude reaches 30-40% of the average value of the signal, while the frequency lies in the range of 0.5-2 Hz. The absolute values of the local Nusselt numbers remain practically the same after the branching along a considerable length of the pipe, and before the start of the transition they exceed the level corresponding to the envelope curve I by 1.2-1.8 times. For the sections with quasilaminar flow in the boundary layer, where a tendency toward stabilization of the coefficients of heat exchange is observed at $Re_d = (10\text{-}160)\cdot 10^3$, the following interpolation formula can be proposed:

$$\mathrm{Nu}_d = 155 - 15.1\exp[-5.8\cdot 10^{-6}\,\mathrm{Re}_d]. \tag{5}$$

For a given Reynolds number the joint solution of Eqs. (4) and (5) allows one to find the coordinate X/d below which one should take Nu_d = const with sufficient accuracy for engineering calculations.

The satisfactory agreement of the experimental data at small X/d with the experimental results of [1], where a Vintoshinskii nozzle with a constriction of 4 was used, leads to the conclusion that the velocity profile and the thickness of the dynamic boundary layer in the

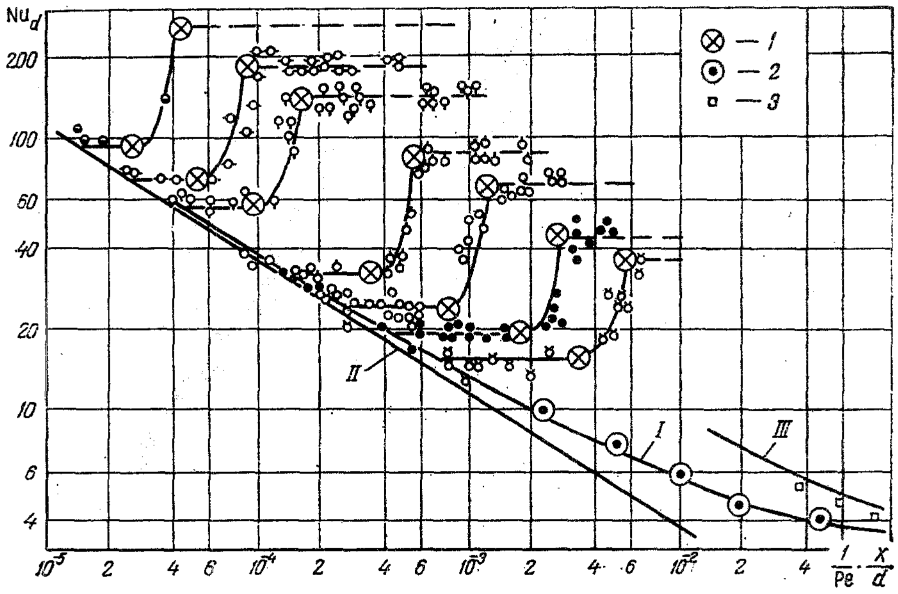

Fig. 4. Analysis of experimental data on local heat exchange in the initial section of a pipe in the coordinates $Nu_d = f(1/Pe \cdot X/d)$. Notation: same as in Fig. 2. Curve I) from Eq. (4); II) from (3); III) from (7); 1) start and end of transition zone; 2) based on data of [10]; 3) based on data of [13].

entrance section of the pipe itself do not belong among the strongly acting factors with respect to the coefficients of heat exchange so long as a mode of Nu_d = const has not set in. This point of view is reinforced by a comparison of the dependence (4) with the results of a numerical solution of the heat-exchange problem for the dynamic initial section of a pipe with $Re_d < 2200$, t_w = const, and developing parabolic velocity distributions [10]. On the right-hand part of Fig. 4 at $1/Pe \cdot X/d > 5 \cdot 10^{-3}$ curve I connects up with the data of the solution of [10] and leads to a limiting Nusselt number $Nu_\infty = 3.66$, when the dimensionless longitudinal coordinate is $1/Pe \cdot X/d = 8 \cdot 10^{-2}$. At $1/Pe \cdot X/d < 5 \cdot 10^{-3}$ the data of the numerical solution of [10] lie 8-10% higher than curve I.

Departures of the Nusselt numbers from the dependence (3) and a tendency toward their stabilization before the sharp rise in the heat-exchange intensity in the transition zone similar to those described above were discovered earlier in [11]. These phenomena were perceived as the start of the transiton. Equation (6), which, like (5), does not contain the longitudinal coordinate, was proposed in [12] for the indicated region with quasilaminar flow and degrees of turbulence of more than 0.6% beyond the nozzle:

$$Nu_d = 0.0164\, Re_d\, \varepsilon^{0.25}. \tag{6}$$

The formal similarity between the configuration of the curves of $Nu_d = f(1/Pe \cdot X/d)$ with $Re_d > 10^4$ and with laminar flow in the boundary layer and the analogous dependence for the region of $Re_d < 2200$ attracts attention. However, the reasons leading to stabilization of the heat exchange at some distance from the entrance have different natures in the two cases. Low-frequency transverse oscillations in velocity, of which the low-frequency oscillations in local heat fluxes mentioned above are a reflection, develop beyond the point of stability loss in the laminar boundary layer under conditions of mixed flow. The mechanism of the formation of low-frequency transverse velocity components in a laminar boundary layer has been traced in detail in the study of Schubauer and Skramstad, the content of which is presented in [14]. It is evident that low-frequency oscillations of velocity in a laminar boundary layer lead to intensification of the heat-exchange process, as a consequence of which the local coefficients of heat exchange hardly decrease in the section from the point of stability loss to the point of the start of the transition. At the point corresponding to the start of the transition the low-frequency oscillations in velocity develop into high-frequency turbulent pulsations and a vigorous increase in the heat-exchange intensity begins.

By integrating Eq. (4) we obtain the dependence for the average coefficients of heat exchange:

$$\mathrm{Nu}_d = 0.76(1/\mathrm{Pe}\cdot X/d)^{-0.5} + 2.3. \qquad (7)$$

In Fig. 4, curve III, calculated from Eq. (7), is compared with the results of measurements made in [13] of the average heat exchange during the laminar flow of air in a pipe with an inner diameter of 3 mm, Reynolds numbers Re_d = 600-3000, and a constant wall temperature. The experimental data lie 9-10% below the calculated data, since the thermal resistance of the side of the heat-transfer surface being heated by the vapor was not taken into account in [13].

NOTATION

X, longitudinal coordinate, reckoned from the start of constriction, m; X_{cr}^{I}, X_{cr}^{II}, coordinates of start and end of transition zone, m; Y, coordinate reckoned along normal to wall, m; R, radius of rounding of entrance nozzle, m; d, inner diameter of pipe, m; δ_W, thickness of dynamic boundary layer, m; α_{loc}, local coefficient of heat transfer, W/m²•deg; t_w, wall temperature, °C; t_{in}, air temperature at entrance to flow section of stand, °C; W, mean flow-rate velocity of stream, m/sec; ε, degree of air turbulence, %; $Re = Wd/\nu$, $Re_X = WX/\nu$, Reynolds numbers referred to pipe diameter and to longitudinal coordinate, respectively; $Re_{Xcr}^{I} = WX_{cr}^{I}/\nu$, $Re_{Xcr}^{II} = WX_{cr}^{II}/\nu$, first and second critical Reynolds numbers, characterizing the position of the transition zone; $Nu_d = \alpha_{loc}d/\lambda$, $Nu_X = \alpha_{loc}X/\lambda$, local Nusselt numbers referred to pipe diameter and to longitudinal coordinate, respectively; $\overline{Nu}_d = \bar{\alpha}d/\lambda$, mean Nusselt number referred to pipe diameter; $Pe = Wd/a$, Peclet number; Pr, Prandtl number.

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